Supplemental Material for the paper untitled "A Robust Method for Strong Rolling Shutter Effects Correction Using Lines with Automatic Feature Selection"

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Abstract

In this supplementary material the reader may find: ***(i)*** *more details about derivations of 3D straight line projection with different rolling shutter (RS) models (section. 1).* ***(ii)*** *Details of practical 4-curves linear solution (R4C) (section. 2).* ***(iii)*** *Besides, we give proofs of degeneracy analysis (section. 3).* ***(iv)*** *We also provide correction results of a RS video by our method.*

1. Details of Parametrization of 3D Line Projection

In order to parameterize 3D straight line projection with different RS models, we first denote components inside intrinsic matrix $\mathbf{K}$ of a calibrated RS camera as:

$$(\mathbf{K}^\top)^{-1} = \begin{vmatrix} f_x & 0 & 0 \\ 0 & f_y & 0 \\ c_x & c_y & 1 \end{vmatrix} \tag{1}$$

Thus, a 2D projected line for global shutter (GS) model in Eq. (6) in the paper can be further expressed as:

$$f_x m_{cip}^x u + f_y m_{cip}^y v + c_x m_{cip}^x + c_y m_{cip}^y + m_{cip}^z \tag{2}$$

where we let the direction vector of straight line as $\mathbf{m}_{cip} = \begin{pmatrix} m_{cip}^x & m_{cip}^y & m_{cip}^z \end{pmatrix}^\top$.

1.1. Deriving 3D Line Projection with Uniform RS Model

The uniform RS model considers both translational and rotational motion during image acquisition. We denote the linear velocity as $\mathbf{d}$ and the angular velocity as $\boldsymbol{\omega}$, and the translation between the v-th row camera frame and world coordinate frame as $\mathbf{t}_c^w + \mathbf{d}v$ while rotation is $(\mathbf{I} + [\boldsymbol{\omega}]_\times v)\mathbf{R}_w^c$. $\mathbf{t}_c^w$ and $\mathbf{R}_w^c$ are translation and rotation matrix for the first row which we set as reference row. Thus, the 3D straight line expression in Eq. (4) in the paper for GS case will change according to uniform RS camera ego-motion:

$$\begin{aligned} \mathbf{R}_c &= ((\mathbf{I} + [\boldsymbol{\omega}]_\times v)\mathbf{R}_w^c)^\top \mathbf{R}_w \\ \mathbf{t}_c = (t_x, t_y, t_z)^\top &= (\mathbf{R}_w)^\top (\mathbf{t}_c^w + \mathbf{d}v) \\ (a_c, b_c) &= (a_w - t_x, b_w - t_y) \end{aligned} \tag{3}$$

In order to make it easier to derived curve expression, we let:

$$\begin{aligned} \mathbf{R}_c = ((\mathbf{I} + [\boldsymbol{\omega}]_\times v)\mathbf{R}_w^c)^\top \mathbf{R}_w = \mathbf{R}_c^w \mathbf{R}_w + \mathbf{R}_c^w [\boldsymbol{\omega}]_\times \mathbf{R}_w v &= \mathbf{A} + \mathbf{B}v \\ a_c = a_w - \mathbf{R}_{w1}^\top \mathbf{t}_c^w - \mathbf{R}_{w1}^\top \mathbf{d}v &= C^a - D^a \\ b_c = b_w - \mathbf{R}_{w2}^\top \mathbf{t}_c^w - \mathbf{R}_{w2}^\top \mathbf{d}v &= C^b - D^b \end{aligned} \tag{4}$$

where, $\mathbf{A}$ and $\mathbf{B}$ are two 3×3 matrices, C^a, C^b, D^a and D^b are scalar variables. Thus, the direction vector of straight line $\mathbf{m}_{cip}$ are now also determined by row-index v:

$$\begin{aligned} m_{cip}^x &= (C^a A_{12} - C^b A_{11}) + (C^a B_{12} - D^a A_{12} - C^b B_{11} + D^b A_{11})v + (D^b B_{11} - D^a B_{12})v^2 \\ m_{cip}^y &= (C^a A_{22} - C^b A_{21}) + (C^a B_{22} - D^a A_{22} - C^b B_{21} + D^b A_{21})v + (D^b B_{21} - D^a B_{22})v^2 \\ m_{cip}^z &= (C^a A_{32} - C^b A_{31}) + (C^a B_{32} - D^a A_{32} - C^b B_{31} + D^b A_{31})v + (D^b B_{31} - D^a B_{32})v^2 \end{aligned} \tag{5}$$

We rewrite Eq. (5) using auxiliary variables L_0^x, L_1^x, L_2^x, L_0^y, L_1^y, L_2^y, L_0^z, L_1^z and L_2^z in oder to do further derivations:

$$\begin{aligned} m_{cip}^x &= L_0^x + L_1^x v + L_2^x v^2 \\ m_{cip}^y &= L_0^y + L_1^y v + L_2^y v^2 \\ m_{cip}^z &= L_0^z + L_1^z v + L_2^z v^2 \end{aligned} \tag{6}$$

Now we substitute $\mathbf{m_{cip}}$ in Eq. (2) by Eq. (5). We can obtain expression of a curve instead of a straight line:

$$\begin{aligned} {}^{Unif}curve(u,v) &= (f_y L_2^y)v^3 + (f_x L_2^x)v^2 u + (f_y L_1^y + c_x L_2^x + c_y L_2^y + L_2^z)v^2 \\ &+ (f_x L_1^x)vu + (f_y L_0^y + c_x L_1^x + c_y L_1^y + L_1^z)v + (f_x L_0^x)u + (c_x L_0^x + c_y L_0^y + L_0^z) \\ &= {}^{Unif} F_1 v^3 + {}^{Unif} F_2 v^2 u + {}^{Unif} F_3 v^2 + {}^{Unif} F_4 vu + {}^{Unif} F_5 v + {}^{Unif} F_6 u + {}^{Unif} F_7 = 0 \end{aligned} \tag{7}$$

There are seven coefficients in Eq. (7): ${}^{Unif}F_1$, ${}^{Unif}F_2$, ${}^{Unif}F_3$, ${}^{Unif}F_4$, ${}^{Unif}F_5$, ${}^{Unif}F_6$ and ${}^{Unif}F_7$.
As with the previous models, when $\mathbf{d}$ and angular velocity $\boldsymbol{\omega}$ equal to zero, Eq. (7) will collapse into Eq. (2) as a GS case.

1.2. Deriving 3D Line Projection with Linear RS Model

Distinctively, if we assume that the angular velocity $\boldsymbol{\omega}$ is equal to zero. Eq. (4) becomes:

$$\begin{aligned} \mathbf{R}_c &= \mathbf{R}_c^w \mathbf{R_w} \\ \mathbf{t}_c = (t_x, t_y, t_z)^\top &= (\mathbf{R}_w)^\top (\mathbf{t}_c^w + \mathbf{d}v) \\ (a_c, b_c) &= (a_w - t_x, b_w - t_y) \end{aligned} \tag{8}$$

Thus, the straight line director vector $\mathbf{m}_{cip}$ can be expressed as follows:

$$\begin{aligned} m_{cip}^x &= ((a_w - \mathbf{R}_{w1}^\top \mathbf{t}_c^w)R_{c12} - (b_w - \mathbf{R}_{w2}^\top \mathbf{t}_c^w)R_{c11}) + (\mathbf{R}_{w2}^\top R_{c22} - \mathbf{R}_{w1}^\top R_{c12})\mathbf{d}v \\ m_{cip}^y &= ((a_w - \mathbf{R}_{w1}^\top \mathbf{t}_c^w)R_{c22} - (b_w - \mathbf{R}_{w2}^\top \mathbf{t}_c^w)R_{c21}) + (\mathbf{R}_{w2}^\top R_{c21} - \mathbf{R}_{w1}^\top R_{c22})\mathbf{d}v \\ m_{cip}^z &= ((a_w - \mathbf{R}_{w1}^\top \mathbf{t}_c^w)R_{c32} - (b_w - \mathbf{R}_{w2}^\top \mathbf{t}_c^w)R_{c31}) + (\mathbf{R}_{w2}^\top R_{c31} - \mathbf{R}_{w1}^\top R_{c32})\mathbf{d}v \end{aligned} \tag{9}$$

We denote Eq. (9) using auxiliary variables L_0^x, L_1^x, L_0^y, L_1^y, L_0^z and L_1^z in order to do further derivations:

$$\begin{aligned} m_{cip}^x &= L_0^x + L_1^x v \\ m_{cip}^y &= L_0^y + L_1^y v \\ m_{cip}^z &= L_0^z + L_1^z v \end{aligned} \tag{10}$$

Now we substitute $\mathbf{m_{cip}}$ in Eq. (2) by Eq. (9). Curves expression in Eq. (7) becomes a hyperbolic curve:

$$\begin{aligned} {}^{Lin}curve(u,v) &= (f_y L_1^y)v^2 + (f_x L_1^x)vu + (f_y L_0^y + c_x L_1^x + c_y L_1^y + L_1^z)v + (f_x L_0^x)u + (f_x L_0^x + c_y L_0^y + L_0^z) \\ &= {}^{Lin} F_1 v^2 + {}^{Lin} F_2 vu + {}^{Lin} F_3 v + {}^{Lin} F_4 u + {}^{Lin} F_5 = 0 \end{aligned} \tag{11}$$

There are five coefficients in Eq. (11): ${}^{Lin}F_1$, ${}^{Lin}F_2$, ${}^{Lin}F_3$, ${}^{Lin}F_4$ and ${}^{Lin}F_5$.

1.3. Deriving 3D Line Projection with Rotate-only RS Model

To obtain rotate-only model, we set linear velocity $\mathbf{d}$ to zero. Thus, Eq. (4) becomes:

$$\begin{aligned}\mathbf{R}_c &= ((\mathbf{I}+[\boldsymbol{\omega}]_\times v)\mathbf{R}_w^c)^\top \mathbf{R}_w \\ \mathbf{t}_c &= (t_x, t_y, t_z)^\top = (\mathbf{R}_w)^\top \mathbf{t}_c^w \\ (a_c, b_c) &= (a_w - t_x, b_w - t_y)\end{aligned} \tag{12}$$

Similarity, we assume:

$$\begin{aligned}\mathbf{R}_c| &= ((\mathbf{I}+[\boldsymbol{\omega}]_\times v)\mathbf{R}_w^c)^\top \mathbf{R}_w \\ &= \mathbf{R}_c^w \mathbf{R}_w + \mathbf{R}_c^w [\boldsymbol{\omega}]_\times \mathbf{R}_w v = \mathbf{A} + \mathbf{B}v\end{aligned} \tag{13}$$

where, $\mathbf{A}$ and $\mathbf{B}$ are two 3×3 matrices. Thus, the direction vector of straight line $\mathbf{m_{cip}}$ is now also determined by row-index v:

$$\begin{aligned}m_{cip}^x &= (a_c A_{12} - b_c A_{11}) + (a_c B_{12} - b_c B_{11})v \\ m_{cip}^y &= (a_c A_{22} - b_c A_{21}) + (a_c B_{22} - b_c B_{21})v \\ m_{cip}^z &= (a_c A_{32} - b_c A_{31}) + (a_c B_{22} - b_c B_{31})v\end{aligned} \tag{14}$$

We denote Eq. (14) using auxiliary variables L_0^x, L_1^x, L_0^y, L_1^y, L_0^z and L_1^z in order to do further derivations:

$$\begin{aligned}m_{cip}^x &= L_0^x + L_1^x v \\ m_{cip}^y &= L_0^y + L_1^y v \\ m_{cip}^z &= L_0^z + L_1^z v\end{aligned} \tag{15}$$

Now we substitute $\mathbf{m_{cip}}$ in Eq. (2) by Eq. (14). Curves expression in Eq. (7) becomes another hyperbolic curve:

$$\begin{aligned}{}^{Rot}curve(u,v) &= (f_y L_1^y)v^2 + (f_x L_1^x)vu + (f_y L_0^y + c_x L_1^x + c_y L_1^y + L_1^z)v + (f_x L_0^x)u + (f_x L_0^x + c_y L_0^y + L_0^z) \\ &= {}^{Rot}F_1 v^2 + {}^{Rot}F_2 vu + {}^{Rot}F_3 v + {}^{Rot}F_4 u + {}^{Rot}F_5 = 0\end{aligned} \tag{16}$$

There are five coefficients in Eq. (16): ${}^{Rot}F_1$, ${}^{Rot}F_2$, ${}^{Rot}F_3$, ${}^{Rot}F_4$ and ${}^{Rot}F_5$.

2. Details of the linear 4-curves Angular Velocity Extraction

2.1. *Proof. Transformation from Eq. (9) to Eq. (10) in the paper.*

We first number equations inside Eq. (10) in the paper and express them using intrinsic parameters defined in Eq. (1).

$$F_1 = f_y(s_1\omega_3 - s_3\omega_1) \tag{17}$$

$$F_2 = f_x(s_3\omega_2 - s_2\omega_3) \tag{18}$$

$$F_3 = f_y s_2 + c_x(s_3\omega_2 - s_2\omega_3) + c_y(s_1\omega_3 - s_3\omega_1) + (s_2\omega_1 - s_1\omega_2) \tag{19}$$

$$F_4 = f_x s_1 \tag{20}$$

$$F_5 = f_x s_1 + c_y s_2 + s_3 \tag{21}$$

In order to extract angular velocity from 17 to 21, we can first substitute structure unknowns s_1, s_2 and s_3 just by angular velocity parameters ω_1, ω_2 and ω_3. Thus, we define four auxiliary variables a, b, c and d:

$$a = F_5 - F_4 = c_y s_2 + s_3 \tag{22}$$

Then we substitute s_2 using 18 and 22, we have:

$$b = \frac{F_2}{f_x} = s_3\omega_2 - \frac{a - s_3}{c_y}\omega_3 \tag{23}$$

Later, we further substitute s_1 using 17 and 20, we have:

$$c = \frac{F_1}{f_y} = \frac{F_4}{f_x}\omega_3 - s_3\omega_1 \tag{24}$$

Besides, we still need to substitute s_1, s_2 and ω_3 using 17,18 and 19:

$$d = F_3 - \frac{c_y}{f_y}F_1 - \frac{c_x}{f_x}F_2 = (f_y + \omega_1)\frac{a - s_3}{c_y} - \frac{F_4}{f_x}\omega_2 \tag{25}$$

Finally, we substitute ω_3 and s_3 in 23, 24 and 25 by ω_1 and ω_2, then we get:

$$c_y[(d - \frac{af_y}{c_y} + \frac{a}{c_y})\omega_1 - \frac{f_yF_4}{f_x}\omega_2 + \frac{F_4}{f_x}\omega_1\omega_2 + \frac{a}{c_y}\omega_1^2 + (\frac{af_y}{c_y} - f_yd)][\omega_2 - \frac{af_x}{F_4c_y}c] = 0 \tag{26}$$

Eq. (26) can be re-written as a cubic bi-varibales polynomial from shown in Eq. (14):

$$C_1{\omega_1}^3 + C_2{\omega_2}^2\omega_1 + C_3{\omega_1}^2 + C_4{\omega_2}^2 + C_5\omega_1\omega_2 + C_6\omega_1 + C_7\omega 2 + C_8 = 0 \tag{27}$$

Where coefficients C_1 to C_8 are:

$$C_1 = -\frac{a^2f_xf_y}{F_4c_y} \tag{28}$$

$$C_2 = -\frac{c_yF_4}{f_x} \tag{29}$$

$$C_3 = (-\frac{af_x}{F_4})(d - \frac{af_y}{c_y} + \frac{a}{c_y}) + \frac{f_xac}{F_4c_y} + a^2 - b - \frac{f_x}{F_4c_y}ac \tag{30}$$

$$C_4 = -\frac{f_yF_4c_y}{f_x} + \frac{F_4^2c_y^2}{f_x^2} \tag{31}$$

$$C_5 = dc_y - af_x + a + f_ya + c + 2aF_4c_y \tag{32}$$

$$C_6 = \frac{af_y}{F_4}(f_yd - a\frac{f_y}{c_y}) + \frac{f_xc}{F_4}(d - a\frac{f_y}{c_y} + \frac{a}{c_y}) + 2adc_y - 2a^2f_y(b + \frac{f_x}{F_4c_y}ac) \tag{33}$$

$$C_7 = (af_y - f_yc_yd) - f_yc + 2\frac{dF_4c_y^2}{f_x} - 2\frac{ac_yF_4f_y}{f_x} \tag{34}$$

$$C_8 = \frac{cf_x}{F_4}(\frac{af_y}{c_y} - f_yd) + c_y^2d^2 - 2ac_yf_yd + a^2f_y^2 - f_y^2(b + \frac{f_x}{F_4c_y}ac) \tag{35}$$

2.2. *Proof. Transformation from Eq. (10) to Eq. (11) in the paper.*

We first re-write the four curves in the form of Eq. (27):

$$C_1^1{\omega_1}^3 + C_2^1{\omega_2}^2\omega_1 + C_3^1{\omega_1}^2 + C_4^1{\omega_2}^2 + C_5^1\omega_1\omega_2 + C_6^1\omega_1 + C_7^1\omega 2 + C_8^1 = 0 \tag{36}$$

$$C_1^2{\omega_1}^3 + C_2^2{\omega_2}^2\omega_1 + C_3^2{\omega_1}^2 + C_4^2{\omega_2}^2 + C_5^2\omega_1\omega_2 + C_6^2\omega_1 + C_7^2\omega 2 + C_8^2 = 0 \tag{37}$$

$$C_1^3{\omega_1}^3 + C_2^3{\omega_2}^2\omega_1 + C_3^3{\omega_1}^2 + C_4^3{\omega_2}^2 + C_5^3\omega_1\omega_2 + C_6^3\omega_1 + C_7^3\omega 2 + C_8^3 = 0 \tag{38}$$

$$C_1^4{\omega_1}^3 + C_2^4{\omega_2}^2\omega_1 + C_3^4{\omega_1}^2 + C_4^4{\omega_2}^2 + C_5^4\omega_1\omega_2 + C_6^4\omega_1 + C_7^4\omega 2 + C_8^4 = 0 \tag{39}$$

Now, we defined three auxiliary variables:

$$r^1 = \frac{C_1^1}{C_1^2} \tag{40}$$

$$r^2 = \frac{C_1^1}{C_1^3} \tag{41}$$

$$r^3 = \frac{C_3^1 - r^1 C_3^2}{C_3^3 - r^2 C_3^2} \tag{42}$$

Then we can substitute ω_2 by ω_1 by using Eq. (36), (37), (38) and auxiliary variables above.

$$Eq.(36) - r^1 Eq.(37) - r^3(Eq.(38) - r^2 Eq.(37)) \tag{43}$$

Then we get:

$$T_1^1\omega_1^2 + T_2^1\omega_2^2 + T_3^1\omega_1\omega_2 + T_4^1\omega_1 + T_5^1\omega_2 + T_6^1 = 0 \tag{44}$$

Coefficients T_1^1 to T_5^1 are calculated by C_1^1 to C_8^3. Then we use Eq. (39) to replace Eq. (38) and under transformation (43). We obtain:

$$T_1^2\omega_1^2 + T_2^2\omega_2^2 + T_3^2\omega_1\omega_2 + T_4^2\omega_1 + T_5^2\omega_2 + T_6^2 = 0 \tag{45}$$

2.3. *Proof. Transformation from Eq. (11) to Eq. (12) in the paper.*

At this stage, ω_2 can be substituted by ω_1 using Eq. (44) and Eq. (45):

$$(H_1, H_2, H_3, H_4, H_5)(\omega_1^4, \omega_1^3, \omega_1^2, \omega_1, 1)^\top = 0 \tag{46}$$

where coefficients H_1 to H_5 are calculated T_1^1 to T_5^2. In such case, two bi-variables cubic polynomial equation turn into a quartic polynomial equation with one unknown. Finally, Eq. (47) can be solved directly with four geometric possible solutions, however, only one is correct. Therefore, we choose the most geometrically consistent value.

3. Degeneracy Analysis

We present derivation details of the three degenerate cases of the linear 4-curves solution.

3.1. *Proof. Degenerate* ***case 1***

We assume a 3D line located within y-z-plan and a camera under an arbitrary ego-rotational along x-axis during acquisition. This leads to the following configuration:

$$\begin{cases} \mathfrak{L} =< \mathbf{R_w}, (a_w, b_w)) >= \{< \mathfrak{R}(\forall x_1, 0, 0), (0, \forall x_2) > \quad |x_1, x_2 \in \mathbb{R}\} \\ \boldsymbol{\omega} = \{[\forall x, 0, 0] \quad |x \in \mathbb{R}\} \end{cases} \tag{47}$$

where $\mathfrak{R}(a, b, c)$ is rotation matrix generated by rotation angles a, b and c along x-y-z axis respectively. Substituting in Eq. (47) into Eq. (13)-(16) with $\mathbf{R}_C^W = \mathbf{I}$ and $\mathbf{t}_C^W = [0; 0; 0]$ Eq. (17)-(21) becomes:

$$\begin{cases} F_1 = 0 \\ F_2 = 0 \\ F_3 = f_y L_0^y = 0 \\ F_4 = L_0^x \\ F_5 = c_x L_0^x + L_0^z \end{cases} \tag{48}$$

The equation above indicates that if an arbitrary 3D line within y-z-plane is observed by a RS camera under ego-rotational along x-axis, no matter magnitude of speed, all of these lines will be projected as the same 2D line $u = c_x/f_x$. In other words, a projected curve can be explained by multiple configurations $\{< \mathbf{R_w}, (a_w, b_w)) >, \boldsymbol{\omega}\}$. Thus, configuration in Eq. (47) is a degenerate one.

3.2. *Proof. Degenerate* ***case 2***

This time, we assume a 3D line located within x-z-plan and camera under arbitrary ego-rotation along y-axis during acquisition. This leads to:

$$\begin{cases} \mathfrak{L} =< \mathbf{R_w}, (a_w, b_w)) >= \{< \Re(0, \forall x_1, 0), (\forall x_2, 0) > \quad |x_1, x_2 \in \mathbb{R}\} \\ \boldsymbol{\omega} = \{[0, \forall x, 0] \quad |x \in \mathbb{R}\} \end{cases} \tag{49}$$

Thus, Eq. (17)-(21) becomes:

$$\begin{cases} F_1 = 0 \\ F_2 = 0 \\ F_3 = f_y L_0^y \\ F_4 = f_x L_0^x = 0 \\ F_5 = c_y L_0^y + L_0^z \end{cases} \tag{50}$$

The equation above indicates that if an arbitrary 3D line within y-z-plane is observed by a RS camera under ego-rotation along y-axis, no matter magnitude of speed, all of these lines will be projected as the 2D lines $v = c_y/f_x$. Therefore, the configuration in Eq. (49) is also a degenerate one.

3.3. *Proof. Degenerate* ***case 3***

We assume a 3D line parallel to x-axis and camera under arbitrary ego-rotational along x-axis during acquisition. It leads to:

$$\begin{cases} \mathfrak{L} =< \mathbf{R_w}, (a_w, b_w)) >= \{< \Re(0, \pi/2, 0), (\forall x_1, \forall x_2) > \quad |x_1, x_2 \in \mathbb{R}\} \\ \boldsymbol{\omega} = \{[\forall x, 0, 0] \quad |x \in \mathbb{R}\} \end{cases} \tag{51}$$

Thus, Eq. (17)-(21) will change to:

$$\begin{cases} F_1 = -f_y b_w \omega_1 \\ F_2 = 0 \\ F_3 = f_y L_0^y - c_y b_w \omega_1 + a_w \omega_1 \\ F_4 = 0 \\ F_5 = c_y L_0^y + L_0^z \end{cases} \tag{52}$$

Equation above indicates that if an arbitrary 3D line parallel to x-axis is observed by a RS camera under ego-rotational along x-axis, no matter magnitude of speed, all of these lines will be projected as horizontal 2D lines in image as $F_1 v^2 + F_3 v + F_5 = 0$. Indeed, each of these lines can be explained by the coupling of a_w, b_w and $\boldsymbol{\omega}$. Therefore, the configuration in Eq.(51) is also a degenerate one.