Rolling Shutter Homography and its Applications *Supplemental Materials*

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In this document, we provide the additional details for the mathematical derivations. We denote an equation in the main paper by using underline. For example, Eq. (1) indicate the first equation in this document while Eq.(1) (with underline) denotes the first equation in the main paper.

1 Derivation of Eq.(20) and Eq.(21)

The expression of $\mathbf{A}_1$ in Eq.(16) is rearranged as shown below to obtain a system of linear equations in $\boldsymbol{\omega}_1$ and $\boldsymbol{d}_1$:

$$\begin{aligned}
\mathbf{A}_1 &= -\mathbf{R}_0[\boldsymbol{\omega}_1]_\times + \mathbf{R}_0\mathbf{d}_1\mathbf{n}_0^\top + \mathbf{t}_0\mathbf{n}_0^\top[\boldsymbol{\omega}_1]_\times \\
&= \underbrace{(-\mathbf{R}_0 + \mathbf{t}_0\mathbf{n}_0^\top)}_{\mathbf{G}}[\boldsymbol{\omega}_1]_\times + \mathbf{R}_0\mathbf{d}_1\mathbf{n}_0^\top \\
&= \begin{bmatrix} (\mathbf{G}^\top)_{(2)}\omega_1^z - (\mathbf{G}^\top)_{(3)}\omega_1^y \\ (-\mathbf{G}^\top)_{(1)}\omega_1^z + (\mathbf{G}^\top)_{(3)}\omega_1^x \\ (\mathbf{G}^\top)_{(1)}\omega_1^y - (\mathbf{G}^\top)_{(2)}\omega_1^x \end{bmatrix}^\top \\
&\quad + \begin{bmatrix} n_0^x\mathbf{R}_{0,(1)}\mathbf{d}_1 & n_0^y\mathbf{R}_{0,(1)}\mathbf{d}_1 & n_0^z\mathbf{R}_{0,(1)}\mathbf{d}_1 \\ n_0^x\mathbf{R}_{0,(2)}\mathbf{d}_1 & n_0^y\mathbf{R}_{0,(2)}\mathbf{d}_1 & n_0^z\mathbf{R}_{0,(2)}\mathbf{d}_1 \\ n_0^x\mathbf{R}_{0,(3)}\mathbf{d}_1 & n_0^y\mathbf{R}_{0,(3)}\mathbf{d}_1 & n_0^z\mathbf{R}_{0,(3)}\mathbf{d}_1 \end{bmatrix} \\
&= \begin{bmatrix} 0 & -G_{13} & G_{12} & n_0^x\mathbf{R}_{0,(1)} \\ G_{13} & 0 & -G_{11} & n_0^y\mathbf{R}_{0,(1)} \\ -G_{12} & G_{11} & 0 & n_0^z\mathbf{R}_{0,(1)} \\ 0 & -G_{23} & G_{22} & n_0^x\mathbf{R}_{0,(2)} \\ G_{23} & 0 & -G_{21} & n_0^y\mathbf{R}_{0,(2)} \\ -G_{22} & G_{21} & 0 & n_0^z\mathbf{R}_{0,(2)} \\ 0 & -G_{33} & G_{32} & n_0^x\mathbf{R}_{0,(3)} \\ G_{33} & 0 & -G_{31} & n_0^y\mathbf{R}_{0,(3)} \\ -G_{32} & G_{31} & 0 & n_0^z\mathbf{R}_{0,(3)} \end{bmatrix} \begin{pmatrix} \omega_1^x \\ \omega_1^y \\ \omega_1^z \\ d_1^x \\ d_1^y \\ d_1^z \end{pmatrix} = \begin{pmatrix} \mathbf{A}_{1,11} \\ \mathbf{A}_{1,12} \\ \mathbf{A}_{1,13} \\ \mathbf{A}_{1,21} \\ \mathbf{A}_{1,22} \\ \mathbf{A}_{1,23} \\ \mathbf{A}_{1,31} \\ \mathbf{A}_{1,32} \\ \mathbf{A}_{1,33} \end{pmatrix}
\end{aligned} \tag{1}$$

Q.E.D.

Similarly, for Eq.(21), the expression of $\mathbf{A}_2$ in Eq.(16) is rearranged as shown below to obtain a system of linear equations in $\boldsymbol{\omega}_2$ and $\boldsymbol{d}_2$:

$$\begin{aligned}
\mathbf{A}_2 &= [\boldsymbol{\omega}_2]_\times\mathbf{R}_0 - \mathbf{d}_2\mathbf{n}_0^\top \\
&= \begin{bmatrix} \mathbf{R}_{0,(3)}\omega_2^y - \mathbf{R}_{0,(2)}\omega_2^z \\ -\mathbf{R}_{0,(3)}\omega_2^x + \mathbf{R}_{0,(1)}\omega_2^z \\ \mathbf{R}_{0,(2)}\omega_2^x - \mathbf{R}_{0,(1)}\omega_2^y \end{bmatrix} - \begin{bmatrix} d_2^x\mathbf{n}_0 \\ d_2^y\mathbf{n}_0 \\ d_2^z\mathbf{n}_0 \end{bmatrix} = \\
&\begin{bmatrix} \mathbf{0} & -\mathbf{R}_{0,(3)} & \mathbf{R}_{0,(2)} \\ \mathbf{R}_{0,(3)} & \mathbf{0} & \mathbf{R}_{0,(1)} \\ -\mathbf{R}_{0,(2)} & \mathbf{R}_{0,(1)} & \mathbf{0} \\ -\mathbf{n}_0 & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & -\mathbf{n}_0 & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & -\mathbf{n}_0 \end{bmatrix}^\top \begin{pmatrix} \omega_2^x \\ \omega_2^y \\ \omega_2^z \\ d_2^x \\ d_2^y \\ d_2^z \end{pmatrix} = \begin{pmatrix} \mathbf{A}_{2,11} \\ \mathbf{A}_{2,12} \\ \mathbf{A}_{2,13} \\ \mathbf{A}_{2,21} \\ \mathbf{A}_{2,22} \\ \mathbf{A}_{2,23} \\ \mathbf{A}_{2,31} \\ \mathbf{A}_{2,32} \\ \mathbf{A}_{2,33} \end{pmatrix}
\end{aligned} \tag{2}$$

Q.E.D.

2 Derivation of Eq.(23)

We first substitute Eq.(13) into Eq.(17):

$$\begin{aligned}
\alpha_i\mathbf{q}'_i &= \mathbf{H}_{RS,i}\mathbf{q}_i \\
&= (\mathbf{H}_{GS} + \mathbf{H}_1 v_i + \mathbf{H}_2 v'_i + \mathbf{H}_3 v_i v'_i + \\
&\quad \mathbf{H}_4 v_i^2 + \mathbf{H}_5 v_i^2 v'_i + \mathbf{H}_6 v_i^3 + \mathbf{H}_7 v_i^3 v'_i)\mathbf{q}_i
\end{aligned} \tag{3}$$

Then we list three equations from each row of Eq. (3) respectively as follows:

$$\begin{aligned}
\alpha_i\mathbf{u}'_i = (&\mathbf{H}_{GS,(1)} + \mathbf{H}_{1,(1)}v_i + \mathbf{H}_{2,(1)}v'_i \\
&+ \mathbf{H}_{3,(1)}v_i v'_i + \quad \mathbf{H}_{4,(1)}v_i^2 + \mathbf{H}_{5,(1)}v_i^2 v'_i \\
&+ \mathbf{H}_{6,(1)}v_i^3 + \mathbf{H}_{7,(1)}v_i^3 v'_i)\mathbf{q}_i
\end{aligned} \tag{4}$$

$$\begin{aligned}
\alpha_i\mathbf{v}'_i = (&\mathbf{H}_{GS,(2)} + \mathbf{H}_{1,(2)}v_i + \mathbf{H}_{2,(2)}v'_i \\
&+ \mathbf{H}_{3,(2)}v_i v'_i + \quad \mathbf{H}_{4,(2)}v_i^2 + \mathbf{H}_{5,(2)}v_i^2 v'_i \\
&+ \mathbf{H}_{6,(2)}v_i^3 + \mathbf{H}_{7,(2)}v_i^3 v'_i)\mathbf{q}_i
\end{aligned} \tag{5}$$

$$\begin{aligned}
\alpha_i = (&\mathbf{H}_{GS,(3)} + \mathbf{H}_{1,(3)}v_i + \mathbf{H}_{2,(3)}v'_i \\
&+ \mathbf{H}_{3,(3)}v_i v'_i + \quad \mathbf{H}_{4,(3)}v_i^2 + \mathbf{H}_{5,(3)}v_i^2 v'_i \\
&+ \mathbf{H}_{6,(3)}v_i^3 + \mathbf{H}_{7,(3)}v_i^3 v'_i)\mathbf{q}_i
\end{aligned} \tag{6}$$

where $\mathbf{H}_{GS,(i)}$ is the i^{th} row of $\mathbf{H}_{GS}$. Now we substitute Eq. (6) into Eq. (5) to eliminate α_i and obtain a quadratic equation w.r.t. v'_i:

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$$
(\underbrace{\begin{bmatrix}\mathbf{H}_{2,(3)}^\top\\ \mathbf{H}_{3,(3)}^\top\\ \mathbf{H}_{5,(3)}^\top\\ \mathbf{H}_{7,(3)}^\top\end{bmatrix}^\top \begin{bmatrix}\mathbf{q}_i\\ \mathbf{q}_i v_i\\ \mathbf{q}_i v_i^2\\ \mathbf{q}_i v_i^3\end{bmatrix})}_{a} {v_i'}^2 + (\underbrace{\begin{bmatrix}\mathbf{H}_{GS,(3)}^\top - \mathbf{H}_{GS,(2)}^\top\\ \mathbf{H}_{1,(3)}^\top - \mathbf{H}_{2,(2)}^\top\\ \mathbf{H}_{4,(3)}^\top - \mathbf{H}_{5,(2)}^\top\\ \mathbf{H}_{6,(3)}^\top - \mathbf{H}_{7,(2)}^\top\end{bmatrix}^\top \begin{bmatrix}\mathbf{q}_i\\ \mathbf{q}_i v_i\\ \mathbf{q}_i v_i^2\\ \mathbf{q}_i v_i^3\end{bmatrix})}_{b} v_i' + \underbrace{\begin{bmatrix}-\mathbf{H}_{GS,(2)}^\top\\ -\mathbf{H}_{1,(2)}^\top\\ -\mathbf{H}_{4,(2)}^\top\\ -\mathbf{H}_{6,(2)}^\top\end{bmatrix}^\top \begin{bmatrix}\mathbf{q}_i\\ \mathbf{q}_i v_i\\ \mathbf{q}_i v_i^2\\ \mathbf{q}_i v_i^3\end{bmatrix}}_{c} = 0 \tag{7}
$$

by naming the coefficients of second degree, first degree and constant terms as a, b and c respectively, we obtain the mapping function from $\mathbf{q}_i$ to v_i' which is described by $\beta(u_i, v_i)$ in Eq.(23) as:

$$
\beta(u_i, v_i) = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \tag{8}
$$

Finally, by substituting Eq. (6) into Eq. (4) to eliminate α_i, we obtain the mapping function from $\mathbf{q}_i$ to u_i' which is described by $\alpha(u_i, v_i)$ in Eq.(23) as:

$$
\begin{aligned}
&\alpha(u_i, v_i) = \frac{d}{e}\\
&\text{where}\\
&d = (\mathbf{H}_{GS,(1)} + \mathbf{H}_{1,(1)} v_i + \mathbf{H}_{2,(1)} \beta(u_i, v_i)\\
&\quad + \mathbf{H}_{3,(1)} v_i \beta(u_i, v_i) + \mathbf{H}_{4,(1)} v_i^2 + \mathbf{H}_{5,(1)} v_i^2 \beta(u_i, v_i)\\
&\quad + \mathbf{H}_{6,(1)} v_i^3 + \mathbf{H}_{7,(1)} v_i^3 \beta(u_i, v_i))\mathbf{q}_i\\
&e = (\mathbf{H}_{GS,(3)} + \mathbf{H}_{1,(3)} v_i + \mathbf{H}_{2,(3)} \beta(u_i, v_i)\\
&\quad + \mathbf{H}_{3,(3)} v_i \beta(u_i, v_i) + \mathbf{H}_{4,(3)} v_i^2 + \mathbf{H}_{5,(3)} v_i^2 \beta(u_i, v_i)\\
&\quad + \mathbf{H}_{6,(3)} v_i^3 + \mathbf{H}_{7,(3)} v_i^3 \beta(u_i, v_i))\mathbf{q}_i
\end{aligned} \tag{9}
$$

Q.E.D.

3 DERIVATION OF EQ.(24)

We first substitute Eq.(16) intoEq.(17):

$$
\alpha_i \mathbf{q'}_i = \mathbf{H}_{RS,i} \mathbf{q}_i \quad = (\mathbf{H}_{GS} + \mathbf{A}_1 v_i + \mathbf{A}_2 v_i')\mathbf{q}_i \tag{10}
$$

Then we list three equations from each row of Eq. (10) respectively as follows:

$$
\alpha_i \mathbf{u'}_i = (\mathbf{H}_{GS,(1)} + \mathbf{A}_{1,(1)} v_i + \mathbf{A}_{2,(1)} v_i')\mathbf{q}_i \tag{11}
$$

$$
\alpha_i \mathbf{v'}_i = (\mathbf{H}_{GS,(2)} + \mathbf{A}_{1,(2)} v_i + \mathbf{A}_{2,(2)} v_i')\mathbf{q}_i \tag{12}
$$

$$
\alpha_i = (\mathbf{H}_{GS,(3)} + \mathbf{A}_{1,(3)} v_i + \mathbf{A}_{2,(3)} v_i')\mathbf{q}_i \tag{13}
$$

where $\mathbf{H}_{GS,(i)}$ is the i^{th} row of $\mathbf{H}_{GS}$. Now we substitute Eq. (13) into Eq. (12) to eliminate α_i and obtain a quadratic equation w.r.t. v_i':

$$
\begin{aligned}
&\underbrace{(\mathbf{A}_{2,(3)} q_i)}_{a} {v_i'}^2 + \underbrace{(\mathbf{H}_{GS,(3)} q_i + \mathbf{A}_{1,(3)} q_i v_i - \mathbf{A}_{2,(2)} q_i)}_{b} v_i'\\
&+ \underbrace{(-\mathbf{H}_{GS,(2)} q_i - \mathbf{A}_{1,(2)} q_i v_i)}_{c} = 0
\end{aligned} \tag{14}
$$

by naming the coefficients of second degree, first degree and constant terms as a, b and c respectively, we obtain the mapping function from $\mathbf{q}_i$ to v_i' which is described by $\beta(u_i, v_i)$ in Eq.(24) as:

$$
\beta(u_i, v_i) = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \tag{15}
$$

Finally, by substituting Eq. (13) into Eq. (11) to eliminate α_i, we obtain the mapping function from $\mathbf{q}_i$ to u_i' which is described by $\alpha(u_i, v_i)$ in Eq.(24) as:

$$
\begin{aligned}
&\alpha(u_i, v_i) = \frac{d}{e}\\
&\text{where}\\
&d = (\mathbf{H}_{GS,(1)} + \mathbf{A}_{1,(1)} v_i + \mathbf{A}_{2,(1)} \beta(u_i, v_i))\mathbf{q}_i\\
&e = (\mathbf{H}_{GS,(3)} + \mathbf{A}_{1,(3)} v_i + \mathbf{A}_{2,(3)} \beta(u_i, v_i))\mathbf{q}_i
\end{aligned} \tag{16}
$$

Q.E.D.